

Web as a tool for proving

Web may critically transform the way we understand proofs. With the use of the Web, proving can be seen as a collaborative activity, involving people with different viewpoints, studies and background. An interesting case study is the Polymath project [6,7]. This project was initiated by Timothy Gowers in 2009, who used the comment functionality of his blog, to investigate the question “can we have collaborative proofs in mathematics?” [8]. The initial problem stated by Gowers as an ‘open’ one, was announced to have been “probably solved” after seven weeks, while more than forty people had contributed to its proof. J.P. van Bendegem argues that the Polymath project has interesting implications for the philosophy of mathematics [5]. Another interesting use of the Web as a tool for proving is Goguen’s Kumo proof assistant [9]. Kumo is a web-based proof assistant, having the following novel features:

- Assists with proofs in first-order hidden logic.
- Generates proof documentation for the web, combining proof browsing with background tutorials and explanations, to enhance proof understanding.
- Supports distributed cooperative proving. Users can send parts of proof to other members in the same group and can receive other or alternative parts of the proof from them. All the parts of a proof are saved in a distributed database, the consistency of which is maintained by Kumo.

The Kumo proof assistant and website generator is part of the Tatami Proof Web data structure. Tatami system integrates formal with informal methods, has an online tutorial capability, runs over the web, and is intended to be useful to ordinary software engineers [3].

We claim that all kinds of web provings can be considered as *proof events* (*Web based proof events*). The concept of proof event was introduced by Goguen [1] as a generalised concept incorporating all kinds of proof – mathematical, dialectical, computer proofs. In [2] an attempt is undertaken to transform this concept into a theoretical framework for the understanding of the proving mathematical activity as an interaction that involves two principal agents – a *prover* and an *interpreter* – and requires *validation* by an appropriate social group. Each web proving is a social spatio-temporal event involving particular people and web components. We can distinguish among various types of web-based proof events depending on the kind of contribution of the participants in the proving.

Bibliography

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